

## Corrigendum

Volume 73, Number 1 (1980), in the article "Abstract Study of Optimality Conditions" by Szymon Dolecki, pp. 24–48:

I. Professor J. M. Borwein of Carnegie–Mellon University pointed out that Theorem 2.4, as stated, is not valid. Indeed, it becomes true, when we allow the functions on  $X$  to admit infinite values; dual multifunctions are defined then on  $\bar{\mathbb{R}}^X$  (extended-real-valued functions on  $X$ ). The same applies to Proposition 5.10.

More generally, let  $F$  be a class of functions on  $X$  and  $\Phi$  a class of functions on  $Y$ . Consider a multifunction  $\Gamma$  from  $Y$  into  $X$ ; the dual multifunction of  $\Gamma$  (with respect to  $F$  and  $\Phi$ ) acts from  $F$  into  $\Phi$ , being defined by

$$\Gamma^\Phi f = \{\varphi \in \Phi: \sup_{y \in \Gamma_x^{-1}} \varphi(y) \leq -f(x), x \in X\}$$

for  $f$  in  $F$ .

On treating  $Y$  and  $X$  as families of (extended-real-valued) functions on  $\Phi$  and  $F$ , respectively, we may talk about  $X$ -duals of multifunctions from  $F$  into  $\Phi$ . Let  $F + \Phi$  denote the family of functions on  $X \times Y$  of the form  $\{f + \varphi, f \in F, \varphi \in \Phi\}$ .

**THEOREM** [1, Prop. 5.4; 2; 3]. *The graph of the  $X$ -dual of the  $\Phi$ -dual of  $\Gamma$  is equal to the  $F + \Phi$ -hull of the graph of  $\Gamma$ :*

$$\mathcal{G}(\Gamma^{\Phi^X}) = \text{co}_{F+\Phi} \mathcal{G}(\Gamma).$$

II. The Lagrangian of Rockafellar (6.10) is not, as stated, the Lagrangian associated with  $(F, \Gamma y_0)$ , but the Lagrangian of the multifunction  $\mathbb{P}_X(\varepsilon_F^{-1} \cap \mathbb{P}_Y)$ ,  $\mathbb{P}$  denoting the respective projection multifunction. The correct statement can be found in [4].

## REFERENCES

1. S. DOLECKI, Abstract optimality theory, Report (1979), Mathematics Research Center, Madison, Wisc.
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3. J. M. BORWEIN, private communication.
4. S. DOLECKI, On perspectives of abstract Lagrangians, in "Proceedings, Meeting on Generalized Lagrangians in Systems Economic Theory, Laxenburg, 1979," to appear.

SZYMON DOLECKI

*Institute of Mathematics  
Polish Academy of Sciences  
Warsaw, Poland  
Submitted by Ky Fan*